An Algorithm for the Interpretation of Jazz Harmony

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1 Introduction

The purpose of this algorithm is to procedurally identify twelve possible interpretations (involving root, chord quality, and extensions) of a given set of notes, generated by using all twelve tones as possible roots. For instance, a possible interpretation for the notes $\{E, G, B, D\}$ is $Cmaj7$ 9 if C is considered to be the root, whereas another interpretation of the same notes would be $Emin7$ with E as the root.

The act of interpreting a set of notes consists of determining which intervals relative to a root are present in the given set (for instance, a ♭3, 5, and ♭7). Once the intervals are known, there is sufficient enough information to construct a musical name for the chord (i.e. $Dmin7$).

2 Interpretation Algorithm Description

This algorithm receives harmony as input, and returns a set of twelve interpretations as output. This approach is based on the fact that for any set of notes, there exists a different interpretation using each of the twelve possible tones as root notes, and therefore there are least twelve ways to interpret a given harmony.

Let a harmony H be defined as some subset of the twelve tones used in Western music. Let an interpretation of such harmony be defined as some root note together with a set of intervals such that each interval has some corresponding note in H.

For instance, one possible interpretation of $H = \{B\flat, D, F, A\}$ is

$$I = \langle B\flat, \{1, 3, 5, 7\}\rangle$$

where the intervals map to notes in H as follows:

$$1 \longmapsto B\flat$$
$$3 \longmapsto D$$
$$5 \longmapsto F$$
$$7 \longmapsto A$$

There are restrictions on these sets of intervals, however, such as there not being more than one of a particular type of interval (i.e. a ♭7 and a 7). The purpose of this algorithm, then, is to generate possible sets of intervals that could be used as interpretations of a given harmony (all roots are used, so choosing which roots to use in interpretations is trivial and not part of the problem).

2.1 Constraints

The constraints on which intervals can exist in the presence of other intervals can be expressed as the following collection of logical implications:

$$
\begin{aligned}
\flat 3 &\implies \neg 3 \\
\flat 7 &\implies \neg 7 \\
\flat 5 &\implies \neg(3 \vee 5 \vee 7) \\
\sharp 5 &\implies \neg(\flat 3 \vee \flat 5 \vee 5 \vee \flat 7) \\
\flat\flat 7 &\implies \flat 3 \wedge \flat 5 \wedge \neg(\flat 7 \vee 7) \\
\sharp 9 &\implies \neg \flat 3 \\
\sharp 11 &\implies \neg \flat 5 \\
\flat 13 &\implies \neg \sharp 5 \\
13 &\implies \neg \flat\flat 7 \\
\sharp 13 &\implies \neg \flat 7
\end{aligned}
$$

where the positive occurrence (i.e. 3) of an interval represents its inclusion in the set of intervals for an interpretation, and a negative occurrence (i.e. $\neg 7$) represents its exclusion. The constraints as a whole can be thought of as the logical AND of each of these implications, and a set of intervals I satisfies the constraints iff the value of this conjunction as a whole is $True$ when evaluated with the intervals in I.

The order in which the decision to include each interval is made *does* in fact matter, and is as follows:

$$\langle 1, 3, \flat 3, 7, \flat 7, 5, \flat 5, \sharp 5, \flat\flat 7, \flat 9, 9, \sharp 9, 11, \sharp 11, \flat 13, 13, \sharp 13 \rangle$$

This allows certain intervals to take precedence over others, which is important, as without this it would be possible to misidentify chord tones as very strange alterations. For instance, in a harmony with two notes, with one 10 semitones above the root and the other 11, the first note might be interpreted as a $\flat 7$. Since this implies there is no natural 7, the second note would *have* to be interpreted as some sort of $\flat 1$ or doubly-sharped 13. This situation can be avoided entirely if the natural 7th is given precedence over the $\flat$7th, as with the above ordering.

2.2 Pseudocode

The pseudocode for the interpreter algorithm is as follows.

Algorithm 1 Jazz Harmony Interpreter

$H \leftarrow$ harmony to interpret
$S \leftarrow$ ordered sequence of all possible identifiable intervals
$T \leftarrow$ ordered chromatic sequence of twelve tones
$O \leftarrow$ set of 12 interpretations; starts empty
for each possible root $r \in T$ **do**
    $I \leftarrow$ set of intervals for this interpretation; starts empty
    for each interval $k \in S$ **do**
        $t \leftarrow$ tone in T an interval of k above r
        if $t \in H$ and adding $k \in I$ satisfies system of constraints **then**
            add interval k to I
    add interpretation $\langle r, I \rangle$ to O

3 Naming Interpretations

For human-readability, it is helpful to convert interpretations into strings consistent with standard musical nomenclature. For instance, the interpretation

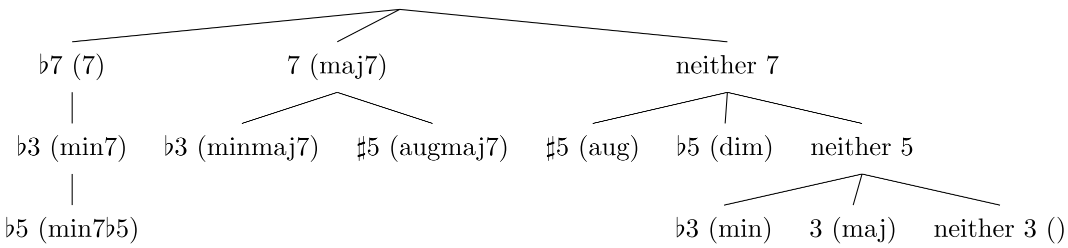
$$\langle C, \{3, 7, \sharp 11\} \rangle$$

would yield the musical name $Cmaj7$ ($\sharp$11)

There are three components of a name of this sort: the root, the chord quality, and the extensions / alterations. Since an interpretation consists of a root and a set of intervals, the root is trivially identified in the interpretation itself. The chord quality and extensions / alterations must be determined from the given set of intervals.

3.1 Chord Quality

Given a set of intervals I, the chord quality of this interpretation can be determined by traversing the following tree as far down as possible, starting at the tree root:



At any given node, proceed to the child node with an interval that is present in I. If none of the intervals listed in child nodes are present, proceed down the "neither" path, or return the chord quality of the current node if possible.

For instance, with $I = \{\flat 3, 7, \sharp 11\}$, the root would be followed to the child node labeled "7 (maj7)", since there is a 7 present in I. From there, the path would be taken to its child node labeled "♭3 (minmaj7)" since there is also a ♭3 in I. This is a leaf node, yielding a final chord quality of "minmaj7" for this interpretation.

3.2 Extensions & Alterations

Extensions and alterations are generally listed after the root and chord quality, so it is sufficient to simply gather all the intervals greater than a 7th in the interpretation and list them. Often, alterations are enclosed in parentheses to avoid confusion of where the sharp or flat is applied.

4 Example

Consider the harmony $H = \{G\flat, B\flat, C, F\}$. By the harmonic interpretation algorithm outlined in Section 2 together with the naming algorithm from Section 3, the following 12 interpretations and their corresponding musical names would be produced:

$$\begin{aligned}
\langle A, \{\flat 3, \flat 9, \flat 13, 13\}\rangle &= Amin\ (\flat 9)\ (\flat 13)\ 13 \\
\langle B\flat, \{1, 5, 9, \flat 13\}\rangle &= B\flat\ 9\ (\flat 13) \\
\langle B, \{7, 5, \flat 9, \sharp 11\}\rangle &= Bmaj7\ (\flat 9)\ (\sharp 11) \\
\langle C, \{1, \flat 7, 11, \sharp 11\}\rangle &= C7\ 11\ (\sharp 11) \\
\langle D\flat, \{3, 7, 11, 13\}\rangle &= D\flat maj7\ 11\ 13 \\
\langle D, \{3, \flat 7, \sharp 9, \flat 13\}\rangle &= D7\ (\sharp 9)\ (\flat 13) \\
\langle E\flat, \{\flat 3, 5, 9, 13\}\rangle &= E\flat min\ 9\ 13 \\
\langle E, \{\sharp 5, \flat 9, 9, \sharp 11\}\rangle &= Eaug\ (\flat 9)\ 9\ (\sharp 11) \\
\langle F, \{1, 5, \flat 9, 11\}\rangle &= F\ (\flat 9)\ 11 \\
\langle G\flat, \{1, 3, 7, \sharp 11\}\rangle &= G\flat maj7\ (\sharp 11) \\
\langle G, \{\flat 3, 7, 11, \sharp 13\}\rangle &= Gminmaj7\ 11\ (\sharp 13) \\
\langle A\flat, \{3, \flat 7, 9, 13\}\rangle &= A\flat 7\ 9\ 13
\end{aligned}$$

5 Application

See the Live MIDI Harmony Interpreter [1] written using this algorithm.

6 Future Work

In the future, a methodology will be developed for scoring these interpretations such that they might be ordered based on the likelihood that the player intended each interpretation. Though this is a highly subjective matter, and mostly based on the tendencies of the individual player, there are certain strategies to explore that involve detracting from interpretations that involve certain extensions that are unlikely to occur over certain chord qualities.

[1] https://github.com/thomascastleman/midi-harmony-interpreter